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Math Assignment 6

17/ENGO4/019

ELECT/ELECT ENGR.

$$u_t - Cu_x = 0$$

$$\Delta x = 0.2 \text{ m}$$

$$u_t = Cu_x$$

Given initial condition,  $u(x, 0) = x^4 \text{ k}$

at  $x = 0 \text{ m}$  &  $x = 1 \text{ m}$

$$u(0, t) = 0 \text{ k} \quad \& \quad u(1, t) = 1 \text{ k}$$

its boundary condition;

At  $t = 0$ , when  $x = 0$

$$u_0 = 0^4 \text{ k}$$

$$= 0$$

when  $x = 0.2 \text{ m}$

$$u_{1,0} = (0.2)^4 \text{ k} = 0.0016 \text{ k}$$

when  $x = 0.4 \text{ m}$

$$u_{2,0} = (0.4)^4 \text{ k} = 0.0256 \text{ k}$$

when  $x = 0.6 \text{ m}$

$$u_{3,0} = (0.6)^4 \text{ k} = 0.1296 \text{ k}$$

when  $x = 0.8 \text{ m}$

$$u_{4,0} = (0.8)^4 \text{ k} = 0.4096 \text{ k}$$

when  $x = 1.0 \text{ m}$

$$u_{5,0} = (1)^4 \text{ k} = 1.000 \text{ k}$$

Using Euler's method.

$$u_{i,j+1} = r u_{i-1,j} + (1-2r) u_{i,j} + r u_{i+1,j}$$

where  $C = 1 \text{ m}^3/\text{day}$   $\Delta t = 0.02 \text{ day}$   $\Delta x = 0.2 \text{ m}$

$$r = \frac{C \Delta t}{\Delta x^2} = \frac{1 \times 0.02}{0.2^2} = 0.5$$

To obtain temperature profile,  $u$

\*  $u$  at  $t = 0.02$

$i = 0$   $i = 1, 2, 3, 4$

at  $i=1$

$$\begin{aligned}U_{1,1} &= rU_{0,0} + (1-2r)U_{1,0} + rU_{2,0} \\ &= 0.5(0) + (1-2(0.5))0.0016 + 0.5(0.0256) \\ &= 0.0128\end{aligned}$$

at  $i=2$

$$\begin{aligned}U_{2,1} &= rU_{1,0} + (1-2r)U_{2,0} + rU_{3,0} \\ &= 0.5(0.0016) + (1-2(0.5))0.0256 + 0.5(0.1296) \\ &= 0.0656\end{aligned}$$

at  $i=3$

$$\begin{aligned}U_{3,1} &= rU_{2,0} + (1-2r)U_{3,0} + rU_{4,0} \\ &= 0.5(0.0256) + (1-2(0.5))0.1296 + 0.5(0.4096) \\ &= 0.2176\end{aligned}$$

at  $i=4$

$$\begin{aligned}U_{4,1} &= rU_{3,0} + (1-2r)U_{4,0} + rU_{5,0} \\ &= 0.5(0.1296) + (1-2(0.5))0.4096 + 0.5(1) \\ &= 0.5648\end{aligned}$$

\*U at  $t=0.04$ ,  $j=1$   $i=1, 2, 3, 4$

at  $i=1$

$$\begin{aligned}U_{1,2} &= rU_{0,1} + (1-2r)U_{1,1} + rU_{2,1} \\ &= 0.5(0) + (1-2(0.5))0.0128 + 0.5(0.0656) \\ &= 0.0328\end{aligned}$$

at  $i=2$

$$\begin{aligned}U_{2,2} &= rU_{1,1} + (1-2r)U_{2,1} + rU_{3,1} \\ &= 0.5(0) + (1-2(0.5))0.0656 + 0.5(0.2176) \\ &= 0.1152\end{aligned}$$

at  $i=3$

$$\begin{aligned}U_{3,2} &= rU_{2,1} + (1-2r)U_3 + rU_{4,1} \\ &= 0.5(0.065) + (1-2(0.5))(0.2176) + 0.5(0.3648) \\ &= 0.3152\end{aligned}$$

at  $i=4$

$$\begin{aligned}U_{4,2} &= rU_{3,1} + (1-2r)U_4 + rU_{5,1} \\ &= 0.5(0.2176) + (1-2(0.5))0.05648 + 0.5(1) \\ &= 0.6088\end{aligned}$$

\*  $U$  at  $t=0.06$  ;  $j=2$   $i=1,2,3,4$

at  $i=1$

$$\begin{aligned}U_{1,3} &= rU_{0,2} + (1-2r)U_{1,2} + rU_{2,2} \\ &= 0.5(0) + (1-2(0.5))0.0328 + 0.5(0.1152) \\ &= 0.0576\end{aligned}$$

at  $i=2$

$$\begin{aligned}U_{2,3} &= rU_{1,2} + (1-2r)U_{2,2} + rU_{3,2} \\ &= 0.5(0.0328) + (1-2(0.5))0.1152 + 0.5(0.3152) \\ &= 0.1740\end{aligned}$$

at  $i=3$

$$\begin{aligned}U_{3,3} &= rU_{2,2} + (1-2r)U_{3,2} + rU_{4,2} \\ &= 0.5(0.1152) + (1-2(0.5))0.3152 + 0.5(0.6088) \\ &= 0.3620\end{aligned}$$

at  $i=4$

$$\begin{aligned}U_{4,3} &= rU_{3,2} + (1-2r)U_{4,2} + rU_{5,2} \\ &= 0.5(0.3152) + (1-2(0.5))(0.6088) + 0.5(1) \\ &= 0.6576\end{aligned}$$

\*  $U$  at  $t=0.08$  ,  $j=3$   $i=1,2,3,4$

at  $i=1$

$$\begin{aligned}U_{1,4} &= rU_{0,3} + (1-2r)U_{1,3} + rU_{2,3} \\ &= 0.5(0) + (1-2(0.5))(0.0576) + 0.5(0.1740) \\ &= 0.087\end{aligned}$$

a at  $i=2$

$$\begin{aligned}U_{2,4} &= rU_{2,3} + (1-2r)U_{2,3} + rU_{3,3} \\ &= 0.5(0.0576) + (1-2(0.5))(0.1740) + 0.5(0.3620) \\ &= 0.2098\end{aligned}$$

at  $i=3$

$$\begin{aligned}\text{at } U_{3,4} &= rU_{2,3} + (1-2r)U_{3,3} + rU_{4,3} \\ &= 0.5(0.1740) + (1-2(0.5))(0.3620) + 0.5(0.6576) \\ &= 0.4158\end{aligned}$$

at  $i=4$

$$\begin{aligned}U_{4,4} &= rU_{3,3} + (1-2r)U_{4,3} + rU_{5,3} \\ \text{at } &= 0.5(0.3620) + (1-2(0.5))(0.6576) + 0.5(1) \\ &= 0.6580\end{aligned}$$

\* U at  $t=0.1$   $j=4$   $i=1,2,3,4$

at  $i=1$

$$\begin{aligned}\text{a } U_{1,5} &= rU_{0,4} + (1-2r)U_{1,4} + rU_{2,4} \\ &= 0.5(0) + (1-2(0.5))(0.087) + 0.5(0.2098) \\ &= 0.1049\end{aligned}$$

at  $i=2$

$$\begin{aligned}\text{* } U_{2,5} &= rU_{1,4} + (1-2r)U_{2,4} + rU_{3,4} \\ &= 0.5(0.087) + (1-2(0.5))(0.4158) + 0.5(0.6576) \\ &= 0.2514\end{aligned}$$

$$\text{at } U_{3,5} = rU_{2,4} + (1-2r)U_{3,4} + rU_{4,4}$$

$$\begin{aligned}&= 0.5(0.2098) + (1-2(0.5))(0.4158) + 0.5(0.651) \\ &= 0.4454\end{aligned}$$

at  $i=4$

$$\begin{aligned}\text{at } U_{4,5} &= rU_{3,4} + (1-2r)U_{4,4} + rU_{5,4} \\ &= 0.5(0.4158) + (1-2(0.5))(0.681) + 0.5(1) \\ &= 0.7079\end{aligned}$$